REMEMBERING FREEMAN DYSON

Brilliant and influential scientist

by *Krishnaswami Alladi*

Freeman Dyson (1923-2020) was a brilliant physicist and mathematician who was influential not only due to his fundamental research contributions, but also because his views on various important scientific issues always attracted worldwide attention. He is known the world over as an outstanding theoretical physicist, but only few outside the realm of mathematics know that he began his research career as an undergraduate by discovering some remarkable properties of the partition function that shed light on some results of the Indian mathematical genius Srinivasa Ramanujan. Both my father and I had the privilege of having known Dyson, and in this article I will provide our personal remembrances. My father's contact with Dyson and his work was from 1949 to the mid-sixties. My correspondence and contact with Dyson was from the early seventies until 2013. I shall begin by giving a brief account of Dyson's work relating to Ramanujan's congruences for the partition function, because this was Dyson's first important discovery, and also because it is this aspect of his research with which I am most familiar. It is also an aspect of Dyson's work that can be easily explained to a non-expert. After this intial brief description of some mathematics, the rest of the article will be non-technical.

Dyson's rank for partitions: By a partition of a positive integer n, we mean a representation of n as a sum of positive integers with the understanding that two such represetations are the same if they differ only in the order in which the parts (= summands) are written. Since order does not matter, we can choose a preferred order, and usually we write the parts in descending order. Thus the partitions of the number 5 are: 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, and 1+1+1+1+1. The number of partitions of an integer n is denoted by $p(n)$, and $p(n)$ is called the partition function; for example $p(5) = 7$ as can be seen from the above list. The theory of partitions was founded by the great Leonard Euler in the eighteenth century who, by the use of generating functions, established elegant results for several partition functions with restrictions on the parts. The theory of partitions has been an active area of research since then.

In the early part of the 20th century, the Indian genius Srinivasa Ramanujan revolutionized the subject by discovering remarkable results involving the partition function. One of his startling discoveries was that

5 divides the number of partitions of an integer of the form $5n + 4$

7 divides the number of partitions of an integer of the form $7n + 5$, and

11 divides the number of partitions of an integer of the form $11n + 6$.

These are the famous *Ramanujan Congruences* for the partition function. Ramanujan's mentor G. H. Hardy of Cambridge University was stunned when Ramanujan stated these congruences, because partitions represent an additive process, and so no one would expect that partitions would satisfy such lovely divisibility relations. Ramanujan gave proofs of the first two of his congruences, but these proofs were analytic in nature. Since partitions are combinatorial objects, it was desirable to understand these congruences combinatorially. Such an explanation was found in 1944 by Freeman Dyson [5] who was an undergraduate mathematics major at Cambridge University at that time.

Dyson defined the *rank of a partition* as the largest part minus the number of parts. For example, the rank of the partition 2+1+1+1 given above is 2-4=-2. Dyson observed that the rank can be used to split the set of partitions of $5n + 4$ into five subsets of equal size, and hence 5 would divide $p(5n + 4)$. Similarly he showed how the rank can be used to split the set of partitions of $7n + 4$ into seven subsets of equal size, and this would imply that 7 divides $p(7n + 4)$. But then he noted that the rank would not explain the third congruence pertaining to 11. He went on to conjecture the existence of a partition statistic that he dubbed the *crank* which would explain why 11 would divide $p(11n + 6)$. Dyson published his findings in a charming paper entitled "Some guesses in the theory of partoitions" in the Cambridge University undergraduate mathematics journal called *Eureka* in 1944. Dyson humorously remarked that it was probably the first instance in mathematics when an object (the crank) was named before it was found! Interestingly, 43 years later, the crank was found by George Andrews and Frank Garvan during the Ramanujan Centennial Conference at the University of Illinois, Urbana, in the summer of 1987 using a vector crank solution to this problem previously given by Garvan in his PhD thesis of 1985. Thus Dyson's conjecture on the crank was resolved four decades later. Since then the study of cranks for general partition functions and their relatives has become an active area of research in number theory.

Dyson's other mathematical work: Dyson made several more fundamental contributions to mathematics. We mention just one more here.

One of the fundamental questions in the study of irrational numbers is to estimate how closely algebraic irrationals can be approximated by rationals. In fact algebraic irrationals cannot be approximated too well by rationals. In 1909, the Norwegian mathematician Axel Thue established a deep result for algebraic numbers of degree at least 3, namely an upper bound on the *irrationality measure* for such algebraic numbers. From this it followed that equations like

$$x^k - dy^k = N$$

where d is a not a k-th power and N any integer, have at most a finite number of solutions in integer values of x and y if the integer k is at least 3. In contrast, such an equation can have infinitely many solutions in integers x and y if $k = 2$. Thue's result was significantly improved by Carl Ludwig Siegel in 1921. Freeman Dyson [6] further improved on Siegel's theorem in 1947, and the final definitive result was established by K. F. Roth in the fifties. Thus Dyson made a notable contribution to this major mathematical problem when he was still a student.

Alladi Ramakrishnan on Freeman Dyson: In 1947, Dyson moved to the United States to earn a PhD in 1949 under the direction of Hans Bethe at Cornell University. There Dyson came into contact with Richard Feynman who simultaneously and independently of Julian Schwinger and Shinichiro Tomonaga had done pioneering work in quantum electrodynamics. But the method of Feynman which was diagramatic was very different from the field theoretic approach of Schwinger and Tomonaga. In 1949 Dyson proved [7], [8], that the two approaches were equivalent and this propelled him to stardom in the world of physics.

In 1949, my father Alladi Ramakrishan had joined the University of Manchester to do a PhD in the area of Probability and Stochastic Processes under the direction of Professor

M. S. Bartlett. But my father's interest extended widely into various parts of physics. In his autobiography, *The Alladi Diary* (World Scientific, 2019) ([1], p. 123) my father says: "In the winter of 1949, I had an opportunity to go to Edinburgh as a participant in the International Conference on Modern Physics with Max Born and Werner Heisenberg as its leading lights. ..Among the physicists I met there was young Freeman Dyson who was just becoming famous..." By the bye, both Dyson and my father were born in 1923, and so were of the same age.

In 1957-58, my parents were in Princeton because my father was a Visiting Member at the Institute for Advanced Study. There he interacted with Dyson. By that time Dyson had become a Permanent Member at the Institute where he remained until his recent demise. The year at the Institute had a transformative effect on my father's academic life in ways more than one. In particular, from 1959, for about a decade, my father worked in elementary particle physics. My father was especially proud of his 1967 paper "Some new topological features of Feynman graphs" [2] in which he showed how the $n!$ Feynman diagrams coalesce in a way that was simpler than Dyson's derivation.

Contacting Dyson in 1972: My fledgeling research in number theory began in 1972 when I was just entering the BSc class at Vivekananda College in Madras. I was only 16 then, and I was fascinated by Fibonacci numbers and arithmetical functions. In order to get an assessment of my early research, my father (who was the Director of MATSCIENCE, The Institute of Mathematical Sciences in Madras) sent my work to very eminent mathematicians to get their opinion and advice. The great Helmut Hasse wrote back saying that he felt my work showed originality and cautioned "let not excessive reading spoil your innate originality." My father also wrote to Dyson because he knew that Dyson began his academic life in number theory as an undergraduate. Dyson wrote back saying that my work showed that I had promise, but that a talented youngster like me should take to a more serious subject like physics, instead of pursuing number theory which he considered "recreational"! Perhaps that is why Dyson moved from number theory to physics. I appreciated that Dyson took time to reply, but I did not follow his advice. I stayed in number theory ever since.

Interacting with Dyson at the Institute for Advanced Study (1981-82): I spent the academic year 1981-82 as a Visiting Member at the Instutute for Advanced Study. One of the daily rituals at the Institute is the Afternoon Tea. It was an occasion to meet other visiting members and to discuss with the permanent members in a relaxed setting. My main mathematical interaction was with the Fields Medallists Atle Selberg and Enrico Bombieri in the School of Mathematics. Dyson was in the School of Natural Sciences, and so I did not see him in the mathematics seminars. But I did see him at Tea where he was mostly surrounded by members of the School of Natural Sciences. I did get to converse with him a few times and he enquired about the work I was doing and the progress I was making.

My wife Mathura and I had a nice apartment - 56 Einstein Drive - on the grounds of the Institute. My daughter Lalitha was just a few months old - she was born in August 1981. Mathura and I were invited to dinners and parties quite a few times, such as at the homes of Harish Chandra and Bombieri, or by Sarvadaman Chowla at a French restaurant in Princeton, or for a party hosted by the Institute Director Harry Woolf. We needed a baby sitter for Lalitha on those occasions when Mathura and I went out in the evenings. I approached Carolyn Underwood, the secretary of the School of Mathematics, for help to find a baby sitter. She said that Dyson's daughter Rebecaa who was 14 years then would be a good baby sitter. So I approached Dyson with this request and he was quite pleased to convey it to his daughter. Indeed, every time Rebecca would babysit Lalitha, Dyson would personally drop his daughter at the front door of our apartment in the evening and pick her up later at night. Each time he would say a warm hello when he dropped her, and a pleasant goodbye when he picked her up.

The Collected/Selected Works of Freeman Dyson: There was an instance when what Dyson said in the Preface to his Collected/Selected Works was very useful to me during my term as Chair of the Mathematics Department at the University of Florida. I was at the CIRM outside of Marseille attending a conference in number theory. At the excellent library of the CIRM, I happened to come across the Collected/Selected Works of Dyson [9]. This book is a collection of ALL his papers in mathematics and only a selection of some of his papers in physics. In the Preface to this book, Dyson says that in mathematics, a theorem proved is a theorem for ever, and so it is customary to publish the collected works of a mathematician. In contrast, in physics, most papers are speculative, and so only years later would the physicist know which papers are truly correct and significant. So it is customary to publish only the selected papers of a physicist. Coincidentally, when I was attending that conference at the CIRM, I received a letter from the Dean saying that the Provost's Tenure and Promotion Committee wanted some justification as to why one of my colleagues was being put up for promotion to full professorship when this person had only 28 research publications instead of the required 30! I pointed out that it is the quality of the publications which should be the main yardstick for evaluation, that this colleague's papers were all in leading journals, and were substantial papers, not very short ones. Quoting Dyson from his Preface, I added a comment that the 28 papers by this colleague would amount to more that 50 by a physicist or a chemist! The promotion was approved without further questioning. By the bye, the Dean and the College Tenure and Promotion Committee had no doubts that this colleague merited the promotion.

80th Anniversary of the Institute for Advanced Study, 2010: My next meeting with Dyson was in Princeton in September 2010 when Mathura and I were invited by Professor Peter Goddard, Director of the Institute for Advanced Study, as his personal guests for the Conference to celebrate the 80th Anniversary of the Institute (see [4]). Goddard is a highly reputed mathematical physicist who was for many years at Cambridge University before taking up the Directorship of the Institute for Advanced Study in 2004. My father knew Goddard well, and so after my father passed away in 2008, Goddard asked me to write an Obituary on my father for the Institute (News) Letter emphasizing how my father's visit to the Institute inspired him to launch the Theoretical Physics Seminar in our family home *Ekamra Nivas* in Madras which ultimately let to the creation of MATSCIENCE in 1962. This was how my contact with Goddard started. Mathura and I were invited to the banquet of the 80th Anniversary Conference. We were honored to be seated next to Professor Goddard and his wife. At the banquet, Freeman Dyson gave a magnificent after dinner speech about the development of theoretical physics at the Institute. In giving a fantastic account of the 80 years of the Institute, Dyson was critical that Robert Oppenheimer, who was the Director from 1947 to 65, concentrated too much on particle physics. Dyson pointed out that it was at his insistence that a program on astrophysics was started at the Institute in 1958 with the appointment of Bengt Stromgren. Dyson had also suggested the great astrophysicist Subramanyam Chandrasekhar of the University of Chicago for a permanent appointment at the Institute, but Chandrasekhar was not interested in the offer, perhaps because he wanted to be close to Yerkes Observatory near Chicago for night sky observations. In retrospect, Dyson said that he felt it was better that Stromgren was appointed as a Permanent Member at the Institute because Chandrasekhar was a "lone wolf" who preferred to work alone and so may not have blended with the culture of the Institute where Permanent Members spend considerable time interacting with the visitors. Dyson is known to be frank and forthright, and what he said in this lecture was a confirmation of this. After the banquet, I chatted with Dyson and told him how much I enjoyed his talk. We reminisced about my father's visit to the Institute in 1957-58, and my visit in 1981-82. Interestingly, the Obituary of my father that I wrote appeared in the Spring 2009 Issue [3] of the Institute Letter, and that same issue contained a feature article on Feynman diagrams, Dyson's work related to this, and a copy of one of Dyson's letters of 1948 to Oppenheimer expressing his disagreement over what Oppenheimer said in the Solway Report.

Visit to Florida in 2013: My last interaction with Dyson was when he visited the University of Florida in March 2013 in reponse to my invitation to deliver the Ramanujan Colloquium. This colloquium series, so generously sponsored by George Andrews, Evan Pugh Professor at The Pennsylvania State University, has enabled us to get world famous mathematicians as speakers every year. Each speaker would give a public lecture of wide appeal, namely the Ramanujan Colloquium, followed by two more specialized seminars during the next two days. The Ramanujan Colloquium is held each year in the Spring Term because that is when Andrews is in residence at the University of Florida.

Dyson arrived in the afternoon of Sunday, March 24. His Ramanujan Colloquium was on Monday at 4:00 pm. Dean Paul D'Anieri, who had a great admiration for Dyson, made the Opening Remarks and attended the lecture in full. Dyson spoke on the theme "Playing with partitions" in which he described how the work of Ramanujan fascinated him, and how he arrived at the notion of the *rank* to combinatorially explain two of Ramanujan's partition congruences. He spoke with enormous energy, something you would not expect in someone who was 90 years of age. After making some introductory comments about Ramanujan and partitions, Dyson surprised (or should I say shocked!) everyone by saying: "I hold Hardy personally responsible for the death of Ramanujan." Dyson pointed out that Ramanujan was away from his family, and that the rigors of life in England during World War II took a toll on his health. He stressed that Ramanujan needed a warm and considerate friend, but Hardy was aloof and did not realize Ramanujan's needs. As another example of Hardy's cold demeanour, he pointed out that when he discovered the rank as an explanation of Ramanujan's congruences, Hardy gave Dyson the cold shoulder and did not show any interest in this work. In thanking Dyson at the end of the talk, I said that we should discuss the Hardy-Ramanujan relationship later in leisure, and we did. My feeling (shared by many) is that Hardy was warm and considerate to Ramanujan, had a special regard for him, and did everything he could not only to get Ramanujan recognized for his work, but also to make his life in England as comfortable as possible under the difficult circumstances.

The large auditorium in which Dyson spoke was packed to capacity. The audience consisted of faculty and students (both graduate and undergraduate) from the mathematics department and other science departments. From the Pennsylvania State University, we had some faculty and undergraduate students of their MASS Program. In the evening following the colloquium, we had a banquet in honor of Dyson at the Hilton Hotel and University Conference Center. It was well attended by many faculty and their spouses. Dean Paul D'Anieri was there with his wife and he paid his tribute to Dyson. James Keesling, my successor as Chair attended the banquet with his wife and gave his message on behalf of the department. In my speech, I referred to my first contact with Dyson in 1972, and reminded him that he advised me then to take to physics instead of number theory. He smiled and nodded when I turned towards him as I said this.

On Tuesday, we took Dyson around campus which looked gorgeous in bright spring time weather. He addressed the Number Theory Seminar that afternoon on the theme "New strategies for prisoner's dilemma". That night, Mathura and I hosted a party in honor of Dyson at our home, which was well attended by many colleagues from the mathematics, physics and statistics departments, and by the students of my graduate number theory class. Dyson was relaxed and conversed with both the faculty and the students.

On Wednesday afternoon, Dyson gave a colloquium in the physics department, as the third of his lectures. The physics department had for many years tried unsucessfully to get Dyson, and so they were thrilled when I called the Chair and offered one of Dyson's talks to be in the physics department. Dyson gave a thrilling talk entitled "Are gravitons in principle detectable?" He started the lecture by saying the following in a thunderous voice: "I hate dogmas and always question them." The physics auditorium was overflowing with many students squatting in the aisles and some standing. Dyson's visit and lectures made a lasting impression on all of us.

A few days after his visit, I wrote a letter thanking him for his lectures and he responded within a few minutes with great warmth. I conclude by appending my letter to him and his response:

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Alladi, Krishnaswami
Sat 3/30/2013 3:28 pm

Dear Professor Dyson,

What a fantastic visit it was! Your three talks (each on a different topic) were magnificent. The physics professors told me that they do not remember when on a previous occasion that hall was filled and overflowing. My abstract algebra students thanked me for suggesting that they should attend your talks. The students in my graduate number theory course appreciated the time you spent in conversation with them during the party at my house. Dean Paul D'Anieri thanked me for the opportunity he had to meet you. On our part, Mathura and I were delighted to have you at our home.

With Andrews' support we are getting truly outstanding speakers for the Ramanujan Colloquium. We are honored to have had you as a speaker in this colloquium series.

Warm regards,

Krishna

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Freeman Dyson < *dyson(at)ias.edu* >
Sat 3/30/2013 4:43 pm

Dear Professor Alladi,

Thankyou for your message, and for your hospitality, and for your efficient organization, and for inviting me to come to celebrate Ramanujan. I enjoyed enormously the meetings with old and new friends, and with students in particular. I had an easy trip home, and today I am back at the Institute dealing with the 214 E-mail messages that arrived during my absence. Thankyou once more for the glorious holiday in Florida.

Yours ever,

Freeman Dyson

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References

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4) Krishnaswami Alladi, “The 80th anniversary of the Institute for Advanced Study”, *http://www.krishnaswami-alladi.com*

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Department of Mathematics, University of Florida, Gainesville, FL 32611, USA
email: alladik(at)ufl.edu