

## REFLECTIONS ON SHREERAM ABHYANKAR

by

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Shreeram Abhyankar (22 July, 1930 - 2 Nov, 2012) was one of world's most eminent algebraic geometers. He ranked among the ten greatest mathematicians of India in the twentieth century. He belonged to the Chitpavan brahmin community of Maharashtra and was proud of its illustrious lineage. After completing his undergraduate studies in India, he went to Harvard University and did his doctoral work there under the direction of Oscar Zariski, one of the most influential figures in algebraic geometry. Abhyankar's PhD thesis on the resolution of singularities problem is a classic, and is among his most important contributions. I was fortunate to get to know him from my boyhood because he was a close friend of my father. Abhyankar and his wife Yvonne were our house guests in India in the sixties. Over the years, we have had several meetings, first in Madras, India, where we hosted him, then in Purdue when my parents and I were his house guests, and finally at the University of Florida where I had the opportunity to host him during my term as Chair. Abhyankar was a fascinating, colorful, and engaging personality. He would grab your attention with his warmth, his open frankness, and his firm opinions on various matters - mathematical and non-mathematical. I have observed him in close quarters, and I will now share a few anecdotes to convey his unusual and engaging personality.

My late father Professor Alladi Ramakrishnan had founded MATSCIENCE, The Institute of Mathematical Sciences, in Madras, India, in 1962, and was its Director until his retirement in 1983 at the age of sixty. The Institute was a realization of a dream of my father and a direct consequence of a Theoretical Physics Seminar that he conducted at our family home in Madras. Abhyankar had a strong attachment to India and a great regard for Indian culture and scientific heritage. Thus he admired my father's efforts in creating such an institute for higher learning and so he visited MATSCIENCE several times in the sixties, his first visit being in August 1963 when he was at Johns Hopkins University, before moving to Purdue. His wife Yvonne always accompanied him and we admired the way she wore the sari - so naturally and elegantly like an Indian lady. During one such visit in January 1968, my father requested Abhyankar to give a Public Lecture on Ramanujan - undoubtedly the greatest mathematician India has ever produced. Out of his profound regard for Ramanujan, Abhyankar readily agreed, even though he was not an expert on the mathematics of Ramanujan. The venue for the lecture was the C. P. Ramaswamy Iyer Foundation, close to our house.

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Sir C. P. Ramaswamy Iyer was a very eminent lawyer, an illustrious statesman, and an orator par-excellence. He was a contemporary and close friend of my grandfather Sir. Alladi Krishnaswami Iyer, who was one of the greatest lawyers of India. After Sir. C. P. died in 1966, a Foundation was created in his name as he desired, and this Foundation held public lectures on the spacious lawns of Sir. C. P.'s mansion called "The Grove". As a twelve year old boy, I attended Abhyankar's brilliant lecture on Ramanujan at the CP Foundation. He held the audience of more than two hundred citizens of Madras in various walks of life in rapt attention as he described some of Ramanujan's most important contributions in his unique and powerful lecturing style. The text of Abhyankar's lecture on Ramanujan appeared in a volume published by the Plenum Press and edited by my father [1].

Abhyankar was proud of his Indian roots and legacy. He valued and emphasized the classical approach to mathematics and did not care for abstraction. He therefore did not agree with the views of the mathematicians of the Tata Institute of Fundamental Research (TIFR) and their complete dedication to the Bourbaki style of doing mathematics and the Grothendieck program in algebraic geometry. He started an institute called *Bhaskaracharya Prathisthana* in his native town of Poona (now Pune) in the state of Maharashtra - an institute inspired by the legacy of the great mathematical guru Bhaskara.

Abhyankar visited the University of Florida regularly from the 1990s to interact with the famous group theorist John Thompson who was Graduate Research Professor in our department. During one such visit, I invited him to address our undergraduate mathematics club  $\pi\mu\epsilon$ . He readily agreed and gave a lovely lecture entitled "An introduction to algebraic geometry". In his talk he stressed that the foundations of algebraic geometry are in classical Cartesian analytic geometry. He lamented that not enough time is spent nowadays in high schools or undergraduate classes to discuss analytic geometry in detail with proofs. He said that his father (also a mathematician), had three years of analytic geometry, but he had only two, and that the younger generation has one year or less on analytic geometry. This decrease in the amount of time spent on analytic geometry worried him. His article "Historical ramblings in algebraic geometry" that appeared in the American Mathematical Monthly in 1976 [2] stresses elementary reasoning in algebraic geometry. His fundamental thesis in this paper is: "The method of high school algebra is powerful. So let us not be overwhelmed by groups-rings-fields or functorial arrows of the other two algebras\*\* and thereby lose sight of the power of the explicit algorithmic process given to us by Newton, Tschirnhausen, Kronecker, and Sylvester." He received the Chauvenet Prize of the MAA in 1978 for this paper.

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\*\*Abhyankar classified algebra into three types: (i) high school algebra (polynomials, power series): Bhaskara (1114), Cardano (1530), Ferrari (1540), Newton (1680), Tschirnhausen (1683), Euler (1748), Sylvester (1840), Cayley (1870), Kronecker (1882), Mertens (1886), König (1903), Perron (1905), Hurwitz (1913), Macaulay (1916), Zariski (1941), Hirronaka (1964), (ii) college algebra (rings, fields, ideals): Dedekind (1882), Neother (1925), Krull (1930), Zariski (1941), Chevalley (1943), Cohen (1946), Nagata (1960), and (iii) university algebra (functors): Serre (1955), Cartan (1956), Eilenberg (1956), Grothendieck (1960), Mumford (1965).

After his talk to  $\pi\mu\epsilon$ , there was a dinner in his honor at my house and there we had a discussion on elementary approaches to deep mathematical problems. I mentioned that in number theory, Paul Erdős was the ultimate champion of the elementary method. To my surprise - or should I say to my shock! - he immediately shot back and said that he did not consider Erdős to be a great mathematician. I asked him why, and he responded saying that Erdős had written some very simple papers. I then said that a mathematician should be judged by his very best work and total contributions and not by his least significant paper. We argued. I gave examples of some ingenious elementary proofs of Erdős following which he asked me how I knew so much about Erdős and his mathematics. I said that Erdős was like a mentor to me, that I had collaborated with him. He immediately exclaimed: “Oh! He is your Guru. Thus I apologize and completely withdraw everything I said because you have every right to defend your Guru, and I should not criticize your Guru in your presence. For example, I would defend my Guru Oscar Zariski against anyone.”

I mentioned earlier that Abhyankar had a great regard for Indian culture. In the Hindu tradition, your Guru is like a God and so should be worshipped. Thus he had unbounded love and respect for Oscar Zariski, his PhD advisor at Harvard. Abhyankar withdrew his arguments not because I was correct about Erdős, but because of his deep respect for the Guru.

Even after my father’s retirement as Director of MATSCIENCE, Abhyankar made it a point to visit our home every time he was in Madras, and call on my parents. After my father died, when I edited a volume in his memory, Abhyankar contributed a massive paper [3] to that volume dedicated both to his father and my father. This paper was originally intended for the Journal of Algebra, but he decided to submit to the volume in memory of my father, and for this I am most grateful.

Even though he was blunt and brutally frank, he was a man on deep feelings and great kindness. He was an eminent mathematician and a fine person.

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## References

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