Four-Dimensional World

Alladi Ramakrishnan, Director, Matscience, Madras.

The nature of space and time has held the fascination of the human mind since the dawn of creation and man's first awareness of the universe around him. Everyone has an intuitive understanding of the three dimensional all-pervasive nature of space and the one-dimensional inexorable flow of time. From the beginning of human thought to the end of the nineteenth century it was axiomatically accepted that space and time are distinct and could be conceived of independent of each other. At the birth of this century, the scientist was faced with one of the greatest paradoxes in the entire history of human knowledge. Contrary to all established notions and normal human intuition it was observationally found that the velocity of light was constant when observed from all frames of reference moving with uniform velocity relative to one another. Every explanation which human ingenuity could invent failed till there came upon the scientific scene an effulgent genius who in one flash enlightened the mystery of light and took his place in the annals of science along with the progenitors of scientific thought like Archimedes, Galileo and Newton. Albert Einstein, using conventional mathematical analysis, formulated an unconventional explanation to solve the baffling paradox. This explanation is strikingly simple as any revelation of undiscovered truth always is. The constancy of the velocity of light implied that the ratio of the spatial interval traversed by light to the time taken was invariant with respect to moving frames of reference. This in turn meant that the difference between the square of the spatial interval and the square of time interval vanished assuming without loss of generality that the velocity of light is set equal to unity.

Einstein made the bold breath-taking assumption that when observed from moving frames of reference both spatial and time intervals traversed by light change in such a way that the difference between their squares is zero. He carried over these considerations to the motion of a massive particle. He postulated that for such a particle the difference though not zero remained

---

negative and constant when observed from various frames of reference. It is well-known in classical theory that the spatial interval changes with the choice of the frame of reference. For example if we are observing a moving particle and take the frame of reference to be the particle itself, the particle is stationary in this frame and the spatial interval becomes zero. Only the time interval remains and this represents the 'age' of the particle. The difference between the square of the zero spatial interval and the square of the time interval is therefore negative. The extraordinary brain-swirling postulate of Einstein consisted in assuming that the time interval characterising the motion of a particle also changes as a function of the spatial interval with the frame of reference. This is without precedent in the history of science or even human knowledge. Once this assumption was made the mathematical transformations which keep the difference invariant could easily be derived and these are known as the Lorentz transformations. In the case of spatial coordinates these just correspond to the classical transformations with an additional factor which becomes increasingly relevant if the velocity of the frame of reference is assumed commensurate with unity. This argument was carried over by Einstein to the time interval which alters with the velocity of the frame of reference. This change is negligible when the velocity is small compared to the unit velocity of light. It led to the physical consequence that no massive particle can attain a velocity greater than that of light. Thus space and time were united into the one-mathematical frame-work and all phenomena studied with reference to a four-dimensional world, the spatial and temporal dimensions being inter-mingled in mathematical transformations.

Once a mathematical postulate is introduced to explain a particular phenomenon, it is logically imperative that all the consequences that flow from that postulate should be interpreted suitably. By a brilliant series of deductions and derivations, Einstein arrived at the famous quadratic relation between energy and momentum of a massive point particle which is just an idealisation of matter. Momentum has three directions corresponding to three-dimensional space while energy is a quantity analogous to time in a transformation theory relating to various frames of reference. The difference between the square of the energy and the square of momentum is constant and just equal to the square of the mass. The immediate logical consequence was the possibility of conversion of matter into energy, a concept which ushered in the nuclear age.

This concept of the physical world had such an impact on human imagination that Einstein's theory was regarded as a total departure from the earlier Newtonian concept in which the separability of space and time was considered sacrosanct.

We shall now present the theory of relativity as the natural continuation or completion of Newtonian ideas which preserves the distinction between space and time and also demonstrates the symmetry under suitable transformations. "The mansion of relativity has many entrances and the most suitable one for entering it from the Newtonian structure is the velocity transformation formula".

This law was deduced by Einstein after obtaining the transformation law for spatial and temporal intervals.

For reasons which will be clear, presently, we shall reverse this historical order and deal with the transformation of velocities as the first and basic concept. We start with the following simple argument which takes us right into the heart of the theory of relativity. Considering one dimensional motion, if v is a possible velocity of a point particle of mass m, so is -v since it merely implies a reversal in direction. Accepting the definition of relative velocity and the axiomatic principle of no preference for any particular frame, we find $2V$ is a realisable velocity. Therefore $2^n V$ is also realisable where n can be chosen as large as we please. If we make the postulate that such a world admitting velocities as large as we please would be 'chaotic', then an upper limit has to be prescribe for the relative velocity and this can be set equal to unity. If V_a and V_b are the velocities of two point particles A and B then the relative velocity V_R is not assumed to be just $V_a - V_b$ as according to the Newtonian concept but has an additional factor which is function of V_a and V_b. *This Einstein factor which can be derived purely on mathematical grounds* acts as a buffer against obtaining relative velocities greater than unity. Now we can define two quantities which are functions of velocity such that their ratio is the velocity of a point particle. Once we know the transformation of velocities, the transformation laws for the functions of velocities follow inevitably. These functions can be so chosen that they have the characteristics of space and time.

The velocity transformation formula connects three quantities each of which is assumed to be is less than unity. Examining carefully the relationship we find it will still be valid for one more case when any two of the three quantities are greater than one and the third is less than unity. It is not

possible to have all the three greater than unity or one of these more than and the other two less than unity. Therefore it remains to interpret the valid relation when two quantities are greater than one one is less than unity.

Such a procedure is not unfamiliar to a mathematician and in fact has been the established more of creating new mathematical structures from existing ones. In such a spirit we interpret the spatial and temporal intervals when their ratio is greater than unity. The velocity concept breaks down but it is quite clear we can interpret them as the spatial and temporal separation of two independent events i.e. they have no causal connection between them. In other words, they refer to *two particles which exist independently of one another.* Since a time difference may exist in some frame of reference we cannot speak of simultaneity of the events but only of independence or acausality. Such an extension of the Lorentz transformation to values of spatial and temporal intervals whose ratio is greater than unity implies the independent existence of two particles, a concept which has to be postulated independently in Newtonian theory but which has been derived as an analytic continuation in the Einstein approach. If matter itself is interpreted as the collection of independently existing point particles distributed in space the existence of matter in bulk seems to be a natural consequence of the acceptance of such intervals. *To identify two points in space we need the independent existence of two particles or in other words we need matter to identify space.* Though matter does not *fill* the whole of space, it can *span* the whole of it when viewed from moving frames of reference. The existence of two independent particles should not be interpreted as events relating to a single particle moving faster than light.

Such misconceptions arise due to the overemphasis on the symmetry of space and time coordinates. It is to be noted that the concept of velocity itself is not symmetrical in space and time, since time is preferred as the denominator and the spatial coordinates as numerators.

We can now summarise the Einstein concept of the universe as follows.

We are aware of the universe around us only through the collection of the realisable events each of which can be characterised by four coordinates i.e. three spatial and one temporal. What we can observe is the space and time intervals between such events. The universe of events falls into two classes:

1. Those separated by time-like intervals i.e. the difference between the squares of the spatial and time intervals is negative. Such events are causally

connected and can be interpreted through the motion of a single massive particle. In a frame of reference in which the particles is at rest the spatial interval vanishes and time interval represents the age of the particle. This flows inexorably and it is impossible to reverse the order of events which are causally connected.

2. Those separated by space-like intervals where the difference between the squares of the spatial and the temporal intervals is positive. i.e. such events are acausal or exist independent of one another. They imply the existence of two and therefore many different identifiable objects separated by space. In fact this is the only meaningful way of identifying the extension of space. These two particles exist simultaneously in a suitable frame of reference. The non-simultaneity of acausal events in other frames does not in any way affect the established concept of the inexorable flow of time relating to causal events.

With such a consistent view of the four-dimensional world the major problem that confronts physicists is to understand how matter is distributed in space and the circumstances under which it can be converted into energy. This amounts to the study of the structure of matter and its extension in space and evolution in time. This is the subject matter of quantum mechanics which is the joint creation of an array of scientists like Planck, Bohr, Rutherford, Heisenberg, Schrodinger and Dirac, in contrast with relativity which is the outcome of a single mind, that of Albert Einstein. At the present time when the triumphs of relativity and quantum mechanics seem to be so complete, the physical world is faced with new characteristics of matter which cannot be understood within the framework of space and time. A significant beginning has been made by Gell-Mann who introduced new attributes to matter and the world awaits another Newton and Einstein to perceive the wonders of God's creation beyond the four-dimensional world.

❧